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Theoretical analysis of nuclear radius measurement using nuclear structure models and Figuretechnology-enhanced computational approaches

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The accurate determination of nuclear radius is fundamental to understanding nuclear structure and interactions. The present study conducts a comprehensive theoretical analysis of nuclear radius measurements using various nuclear structure models, including the empirical mass-number scaling model, the Hartree-Fock approach, and the relativistic mean-field (RMF) theory. These models are systematically compared against experimental nuclear radii to evaluate their predictive accuracy and assess their strengths and limitations. The study also incorporates an uncertainty analysis to quantify the reliability of theoretical predictions, employing Monte Carlo simulations and Bayesian inference techniques to refine estimations. The results reveal that while empirical models provide reasonable approximations, they lack the precision required for heavy nuclei due to the omission of interaction effects. The Hartree-Fock and RMF models incorporate nucleon-nucleon interactions and relativistic corrections, improving predictive performance, yet systematic deviations persist, particularly in neutron-rich nuclei. Comparisons with recent studies highlight the growing role of machine learning techniques in refining nuclear radius predictions, reducing uncertainty margins, and improving model accuracy. The study emphasizes the necessity for hybrid methodologies integrating empirical models, quantum mechanical calculations, and advanced computational techniques to enhance nuclear radius predictions. In addition, Figuretechnology-inspired computational techniques, including Figurescale modeling and machine learning algorithms, offer enhanced predictive capabilities by capturing complex nuclear interactions at finer scales and reducing uncertainty in nuclear radius estimation.

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1. INTRODUCTION

The atomic nucleus, composed of protons and neutrons, is bound by strong nuclear forces that counteract electrostatic repulsion, ensuring matter stability [1]. These forces, one of the four fundamental interactions, govern nuclear structure, stability, and reactions in competition with electromagnetic, weak, and gravitational forces [2]. The shell model explains nuclear structure through discrete nucleon energy levels shaped by effective nucleon–nucleon interactions, affecting spin, binding energy, and excitations [3]. Nuclear density distribution determines size, isotopic shifts, and deformations, requiring precise calculations for accurate nuclear property predictions [4]. High-precision nuclear mass tables from deformed relativistic Hartree–Bogoliubov theory improve modeling of shell structures, especially in exotic nuclei [5]. Advances in nuclear shape imaging via high-energy collisions probe density distributions in real time, revealing that at high densities, short-range interactions shift from tensor to scalar forces—critical for modeling dense environments like neutron stars [6,7]. Quantum chromodynamics further refines understanding by describing nucleon interactions through quark–gluon dynamics, impacting stability, decay, and fusion [8]. Large-scale shell model calculations clarify valence nucleon behavior and excitations [9], while geometric models link electron shell structure and nuclear stability, showing anisotropic forces' role in defining the stability corridor [10]. Nuclear radius studies began with Rutherford's gold foil experiment, identifying a dense nucleus and estimating its size [11]. Later, electron scattering and muonic X-ray spectroscopy refined measurements and charge distribution models [12]. Theoretical approaches evolved with quantum mechanics and density functional theory [12]. Recently, machine learning has accurately predicted nuclear charge radii across isotopic chains [13]. A major challenge in nuclear physics is understanding core swelling in neutron-rich nuclei, where nuclear density saturation causes unexpected radius variations [14]. High-precision laser spectroscopy now enables direct charge radius measurements in exotic, short-lived isotopes [15], while studies of nuclear deformation reveal how density profiles affect radius evolution in light- and medium-mass neutron-rich nuclei [14]. Bayesian neural networks have improved charge radius predictions, especially for isotopes with poorly understood structures [16]. However, proton radius discrepancies persist, fueling debate and investigations into quantum electrodynamics effects on charge radii [17]. This work presents a comparative theoretical analysis of nuclear radius measurements across multiple nuclear structure models, assessing predictive accuracy and the influence of nuclear interaction potentials, density distributions, and force models on calculated radii. Computational simulations compared theoretical predictions to experimental data to evaluate consistency and reliability. Uncertainty analysis considered parameter variations, approximations, and the roles of central, spin–orbit, and tensor interactions in shaping nuclear size. The study also examined scaling laws linking nuclear size to mass number, validating and refining empirical formulas, and extended the scope to charge radius distributions for a broader characterization. By comparing models, the research aimed to refine nuclear structure theories, assess applicability to diverse isotopes, and support advances in both theory and experiment. Nuclear sizes and density distributions are closely linked to proton and neutron wavefunctions [18–19]. Recent advances in Figure technology have significantly influenced computational physics and nuclear modeling. Figure scale modeling techniques and machine learning approaches, inspired by Figure technology concepts, enable the analysis of complex systems with high precision and computational efficiency. These methods allow for improved representation of nuclear density distributions and interaction potentials at very fine scales. In nuclear physics, Figure technology-inspired algorithms can enhance predictive models by reducing uncertainties and capturing nonlinear relationships between

nuclear parameters. Therefore, integrating Figure technology concepts with traditional nuclear structure models provides a powerful framework for improving nuclear radius predictions.

2. METHODOLOGY

2.1 Selection of nuclear structure models

The selection of nuclear structure models is based on their accuracy in describing nucleon distribution and nuclear radii, computational feasibility, and relevance to radius measurement studies. Multiple frameworks are used to capture different aspects of nuclear interactions. The liquid drop model is chosen for its simplicity and ability to approximate radii through mass-number scaling, providing a quick empirical reference despite lacking quantum detail. The shell model incorporated quantum effects by treating nucleons in discrete energy levels within a nuclear potential, refining predictions for magic-number nuclei, though heavy nuclei required complex computational methods. Mean field approaches (Hartree–Fock and Skyrme Hartree–Fock) described interactions via effective nucleon–nucleon forces, solving the many-body problem iteratively to obtain density distributions and radii. The Skyrme variant is favored for its ability to model both spherical and deformed nuclei and its wide use in nuclear structure studies. Relativistic mean field (RMF) models extended the mean field framework by including Lorentz-covariant equations and meson-exchange interactions, enabling accurate comparisons of relativistic vs. non-relativistic predictions, especially for neutron-rich nuclei. Density functional theory (DFT) offered computational efficiency and predictive accuracy by deriving radii from energy density functionals calibrated to experimental data, making it ideal for studying systematic trends across isotopes. This combination of empirical and advanced theoretical models enabled a comprehensive assessment of predictive capabilities, providing diverse perspectives to enhance understanding of nuclear radii [20].

2.2 Selection of model parameters and calibration

Accurate nuclear radius predictions depend on careful calibration of model parameters, as different nuclear structure models use varying approximations and interaction potentials. Parameters are adjusted to fit to experimental data and theoretical considerations to make them agree with known nuclear properties. In the liquid drop model, the proportionality constant in the mass-number scaling law is determined using the data on stable nuclei so as to reproduce the observed sizes, and provided simple and reliable trend predictions with no microscopic basis. In HartreeFock and Skyrme HartreeFock theories, binding energies, charge radii, and density distributions are globally fitted using Skyrme-type forces with coupling constants, and could be used to make consistent predictions across isotopic chains. Relativistic mean field (RMF) models had their coupling constants to scalar and vector fields calibrated to give the correct saturation density, binding energy per nucleon and neutron skin thickness, and with these models, improved predictions of neutron-rich nuclei could be made where relativistic effects are dominant. In density functional theory (DFT), parameters of energy functionals are optimized using charge radii, neutron distributions, and nuclear masses, balancing computational efficiency with predictive accuracy for large-scale studies. Calibration also employed Monte Carlo simulations and Bayesian inference to quantify parameter uncertainties, producing probability distributions and assessing prediction robustness. Sensitivity analyses examined the effect of parameter variations on radii. Through systematic calibration, theoretical predictions are aligned with experimental data, improving model reliability and predictive power, particularly for isotopes lacking direct measurements [21].

2.3 Computational techniques for radius prediction

Predictions of nuclear radii relied on computational methods to solve nuclear structure equations, with complexity varying from analytical solutions to self-consistent numerical techniques. Given the many-body nature of nuclei, such methods are essential for evaluating radii across isotopic chains. In liquid drop and empirical models, radii are obtained directly from algebraic mass-number scaling laws, requiring minimal computation but omitting microscopic effects such as shell structure and correlations. For quantum mechanical approaches, the Schrödinger equation for nucleons in an effective potential is solved numerically. Hartree–Fock methods used variational, iterative solutions on spatial grids or via basis expansions, yielding density distributions from which rms radii are derived. In Skyrme–Hartree–Fock, energy density functionals are optimized through large-scale iterative minimization, incorporating deformations and pairing interactions via finite difference schemes, spectral expansions, and matrix diagonalization. Relativistic mean field (RMF) models solved the Dirac equation with meson-exchange interactions using finite element and relaxation algorithms. Parallel computing enabled large-scale simulations, improving predictions for neutron-rich and heavy nuclei. Monte Carlo simulations generated ensembles of nuclear configurations to assess uncertainties, while Bayesian inference refined predictions by integrating prior experimental data, quantifying confidence intervals and sensitivities. Computational techniques also enabled visualization of nuclear density distributions, allowing direct comparison with experimental electron scattering and charge radius data. Machine learning tools further analyzed nuclear structure patterns, improving predictive accuracy. This integration of advanced numerical methods with high-performance computing ensured precise, systematic, and experimentally consistent evaluations of nuclear radii [22].

2.4 Statistical methods for model comparison

Statistical techniques are inevitable in order to make objective comparisons between the existing nuclear structure models and nuclear radius predictions. Root mean square deviation (RMSD) is used to measure the average discrepancy between the experimental and the predicted radii, which ranked the models according to the extent of agreement. Goodness-of-fit is tested using chi-squared tests that compared squared residuals to expected variances with lower values representing better consistency.

Bayesian modelling used prior information, revising the estimate of parameters based on experimental data and comparing models based on the marginal likelihood, a trade-off between accurate and parsimonious models. Cross-validation is used to compare predictive power by training models on one dataset and validating on unseen data, which minimized overfitting and made it more robust across isotopes. Model complexity is penalized using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), favoring models with lower scores that balanced predictive accuracy and simplicity. Monte Carlo uncertainty analysis generated ensembles of predictions from random parameter sets, producing probability distributions for radii, estimating confidence intervals, and identifying parameter regions causing major discrepancies.

By integrating these statistical tools, the study provided a rigorous, systematic evaluation of model strengths and weaknesses, guiding the refinement of theoretical approaches toward more reliable nuclear radius predictions [23].

2.5 Theoretical predictions of nuclear radii

Nuclear radii have been predicted to the power of multiple theoretical models, empirical mass number scaling law and Hartree Fock approach and RMF model. The calculated results are compared with

experimentally observed nuclear radii for few isotopes to see if the predicted theoretical values are accurate and reliable.
Using the standard mass number scaling formula with nuclear radii estimated empirically given by Eq. (1):

$$R = R_0 . A^{1/3} \quad (1)$$

R_0 = 1.2 fm is the empirical nuclear radius constant and A is the mass number. The model described above is a first order approximation of nuclear size and could not be used to include nucleon interactions, or quantum effects. Finally, self-consistent nucleon interactions are included in the Hartree Fock approach and small corrections to the radius estimates are obtained. As with relativistic mean field model relativistic effects are included by adjusting the predicted radii in terms of meson exchange interactions. In the table the presentation of these models in comparison with experimental nuclear radii for the selected isotopes. Refined estimates from the Hartree Fock and RMF models generally improved agreement to experiment. Nevertheless, further discrepancies remained at heavier nuclei and more corrections might be required to achieve greater accuracy [24].

2.6 Model comparisons and validation

The predictive accuracy of the empirical, Hartree–Fock, and relativistic mean field (RMF) models is evaluated against experimental nuclear radii using absolute errors and root mean square deviation (RMSD) as performance metrics. The empirical model, based on mass-number scaling, provided reasonable size estimates but deviated significantly for heavy nuclei where quantum and interaction effects dominate, yielding RMSD = 0.98 fm. The Hartree–Fock model incorporated self-consistent nucleon interactions, achieving more precise radii than the empirical approach, though accuracy varied by isotope. Deviations are slightly larger than expected (RMSD = 1.16 fm), suggesting a need for improved surface-density treatments. The RMF model, including meson-exchange interactions and relativistic corrections, generally matched experimental values better than Hartree–Fock (RMSD = 1.13 fm), though deviations persisted for some isotopes. Parameter optimization could further improve accuracy. Overall, all models captured general trends but none achieved perfect agreement. The empirical model offers quick first-order estimates, while Hartree–Fock and RMF improve precision by including nucleon interactions, albeit with residual discrepancies (Figure 1) [25].

2.7 Uncertainty analysis

A determination of the uncertainty in nuclear radius prediction is made by considering parameter variation and its effect on calculated nuclear radii. The model parameters are varied at random within a range that is realistic a priori but informed by empirical data and theoretical constraints, using Monte Carlo simulations. A quantitative measure associated with each model is defined by the standard deviation of the predicted nuclear radii.
Using the standard deviation formula, the uncertainty of the nuclear radius ΔR is calculated Eq. (2).

$$\Delta R = \sqrt{\frac{1}{N}\sum(R_i - R_{mean})^2} \quad (2)$$

where individual radius prediction for a given parameter set is denoted as Ri. The average predicted radius averaged over all Monte Carlo samples is given by Rmean. The total number of simulations is denoted as N. The mass-number scaling law is also observed in the radius for the empirical model Eq. (3).

$$R = R_0 . A^{1/3} \quad (3)$$

Such departures in predicted radii are due to the experimental uncertainties in R0. Estimates for the uncertainty in the empirical model is propagated by error in R0 using partial derivatives Eq. (4).

$$\Delta R = \left|\frac{dR}{dR_0}\right| . \Delta R_{0=} R_0 . A^{1/3} \quad (4)$$

Uncertainties of the nuclear interaction parameters are the origin of uncertainty for the models based on Hartree-Fock and relativistic mean field. Using a self-consistent approach, small changes in interaction strengths resulted in changes of the final predicted radius. These perturbations are simulated using Monte Carlo, and radius distributions are produced from which the standard deviation is extracted (Figure 2) [26].

2.8 Impact of nuclear forces on radius predictions

To study the effect of nuclear forces, corrections resulting from central nuclear forces, spin orbit coupling and tensor forces are incorporated into the theoretical models. These forces are a first rank, a first order ingredient in the determination of nuclear structure, and had measurable effects upon the size of nuclei.
The first calculation of the nuclear radius is based on the empirical scaling law Eq. (5):

$$R = R_0 . A^{1/3} \quad (5)$$

R_0 is the empirical constant equal typically to 1.2 fm and A is the nuclear mass number. This gave a crude estimate, but no regard is made for internal nucleon interactions. In this respect, the hartree–fock (HF) and relativistic mean field (RMF) models further improved upon this, by making use of self-consistent nuclear interactions. But those did not explain why the radii are corrected; nuclear forces had to be taken into account to do so. It is computed as the total correction Eq. (6).

$$\Delta R_{total} = \Delta R_{central} + \Delta R_{spin-orbit} + \Delta R_{tensor} \quad (6)$$

where correction due to central nuclear forces is ΔRcentral. The spin-orbit contribution is taken into account by ΔRspin-orbit. The effect of tensor forces is taken into account by ΔRtensor. Then, these force contributions are added to the original theoretical radius and the corrected nuclear radius is obtained Eq. (7).

$$\Delta R_{corrected} = \Delta R_{HF/RMF} + \Delta R_{total} \quad (7)$$

The corrected and uncorrected nuclear Radii for Hartree-Fock as well as RMF are shown in the table below alongside the experimental values [27].

2.9 Figure technology-based computational enhancement

In addition to conventional nuclear structure models, Figure technology-inspired computational methods are considered to enhance prediction accuracy. These approaches include machine learning algorithms and Figure scale simulation techniques that model nuclear interactions with higher resolution. Machine learning models, such as neural networks and Bayesian optimization techniques, can be trained using experimental nuclear data to predict nuclear radii with improved accuracy. These

models capture complex nonlinear relationships that are difficult to represent using traditional analytical methods. Furthermore, Figure scale modeling techniques allow better representation of nuclear density distributions, improving the estimation of nuclear size. Although the present study focuses on theoretical models, the integration of Figure technology-based computational tools is proposed as a future direction to refine nuclear radius predictions and reduce uncertainties.

3. THEORETICAL FRAMEWORK

3.1 Nuclear structure models

Nuclear properties—including size, shape, and internal interactions—are described by various nuclear structure models based on quantum mechanics and many-body physics, each targeting specific aspects of nuclear behavior with varying complexity and predictive accuracy. The liquid drop model treats the nucleus as a continuous nucleon fluid bound by nuclear forces, successfully explaining macroscopic properties such as binding energy and fission, and providing a simple mass-number relation for radius. However, it omits shell and finer quantum effects. The shell model views nucleons as moving in discrete energy levels analogous to atomic orbitals, explaining magic numbers for particularly stable configurations. By incorporating nucleon distributions in potential wells, it enables more precise radius calculations, though it is computationally demanding and limited mainly to heavier nuclei. Self-consistent mean field models, including Hartree–Fock and Skyrme Hartree–Fock, derive nuclear density distributions from effective nucleon–nucleon interactions, solving equations iteratively to determine radii, shapes, and deformations across isotopes. Relativistic mean field models, which are models of relativistic quantum mechanics, add meson-exchange forces and relativistic effects, and are better at predicting saturation properties and radii of neutron-rich and exotic nuclei. Density functional theories (DFT) calculate radii using energy density functionals, and are a trade between computational efficiency and accuracy. Using experimental data to fit functional parameters, DFT can be used to reliably predict nuclear size variations throughout isotopic chains. The models are complementary in their contributions--some aiming at simplicity, others at high accuracy with complex self-consistent calculations. This comparative analysis aids the assessment of their strengths and weaknesses and leads to the improvement of nuclear structure theories.

3.2 Fundamental theories of nuclear radius measurement

Determination of the nuclear radius relates to the theories of the nucleon distribution to the nuclear size that are mathematically formulated by using quantum mechanics, statistical physics, and nuclear interactions. One of the first is the empirical scaling rule between radius and the cube root of the mass number, with uniform nucleon density. This gave crude estimates and did not consider shell effects and deformations. The Fermi gas model described nucleons as free fermions in a potential well, and explained some size properties, but not well interactions and correlations. Higher order HartreeFock treatments used variational principles to effective nuclear potentials, resulting in better rms radii of self-consistent density distributions. Relativistic mean field (RMF) theory introduced the effects of meson exchange and relativistic effects, and better predicted neutron-rich and exotic nuclei, where non-relativistic models are inadequate. The collective model incorporated nuclear vibrations and rotations, and demonstrated that deformation had a significant effect on the radii and could explain size changes along isotopic chains. Density functional theory (DFT), which is based on energy density functionals, provided a reasonable compromise between computational efficiency and accuracy, and systematic radius studies across isotopes could be made when parameters are optimized to data. The role of nuclear forces, the shell structure and collective effects in size determination are explained by each framework, empirical formulas provided the faster estimates, whereas quantum mechanical and relativistic models provided the most accurate and physics-based estimates. The combination of

approaches increased the consistency with experimental data and better predictive work on nuclear structure.

3.3 Governing equations and mathematical formulations

The radius of the nucleus can be calculated theoretically using fundamental equations that describe the spatial distribution of nucleons within the nucleus. Empirical relations, quantum-mechanical principles, and relativistic corrections are used to formulate nuclear size through different mathematical expressions. These equations provide the theoretical basis for predicting nuclear radii and understanding their dependence on nuclear structure models. Assuming that the nuclear density is nearly constant, one of the most widely used empirical expressions for the nuclear radius is given by Eq. (8): In this equation, R is the nuclear radius, R_0 is the proportionality constant of approximately 1.2 fm, and A is the mass number. This relation is derived from the liquid-drop model and provides a first-order description of nuclear size.

$$\Delta R_{=} R_0 . A^{1/3} \tag{8}$$

For a more accurate description, the nuclear root-mean-square radius is used because it accounts for the spatial distribution of nucleons inside the nucleus. The rms radius is defined by Eq. (9): Here, N is the total number of nucleons, and r_i is the position of the *i*th nucleon measured from the center of the nucleus. This formulation is especially useful in theoretical calculations because it reflects the actual distribution of matter rather than assuming a perfectly uniform density.

$$R_{rms} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} r_i^2} \tag{9}$$

The nuclear density distribution is often approximated by a Fermi-type function, also known as the two-parameter Fermi distribution, as given in Eq. (10): In this expression, ρ(r) is the nuclear density at a distance r from the nuclear center, ρ_0 is the central nuclear density, R is the half-density radius, and a is the diffuseness parameter, which describes how smoothly the density decreases from the interior of the nucleus toward the surface. This form is widely used in self-consistent mean-field calculations because it provides a realistic description of nucleon distributions.

$$\rho(r) = \frac{\rho_0}{1+e^{\frac{r-R}{a}}} \tag{10}$$

Within Hartree–Fock calculations, the nuclear radius is obtained from the self-consistent solution of the Schrödinger equation for the single-particle wave functions φ_i, as expressed in Eq. (11). In this equation, H is the nuclear Hamiltonian, φ_i is the wave function of the *i*th nucleon, and E_i is the corresponding energy eigenvalue. The resulting nucleon density distribution is then used to determine the nuclear radius.

$$H . \varphi_i = E_i . \varphi_i \tag{11}$$

In the relativistic mean-field framework, the nuclear radius is calculated from the nucleon density distribution obtained through self-consistent solutions of the Dirac equation, given in Eq. (12): Here, ψ is the nucleon wave function, γ^μ are the Dirac matrices, m is the nucleon mass, and gσ, gω, and gρ are the coupling constants associated with meson–nucleon interactions. This approach incorporates relativistic effects and is particularly important for describing medium and heavy nuclei.

$$\Psi = 0[i.Y^{\mu}.\partial_{\mu} - m - g_{\sigma}.\sigma - g_{\omega}.\gamma^{\mu}.\omega_{\mu}\text{-}g_{\rho}.\gamma^{\mu}.\tau_{a}.\rho_{\mu}^{a} \quad (12)$$

The uncertainty in nuclear-radius predictions is evaluated statistically using Monte Carlo simulations. The uncertainty ΔR is calculated using Eq. (13): In this equation, R_i represents an individual radius prediction from a given parameter set, R_mean represents the mean radius obtained from all simulations, and N is the total number of simulations. This method quantifies the sensitivity of nuclear-radius predictions to variations in model parameters**.**

$$R = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(R_i - R_{mean})^2} \quad (13)$$

4. RESULTS AND DISCUSSION

The accurate theoretical determination of nuclear radii remains essential for understanding nuclear structure, density distributions, and the role of nucleon–nucleon interactions across isotopic systems. In the present work, nuclear radius predictions are examined using complementary theoretical frameworks, including the empirical mass-number scaling relation, the Hartree–Fock approach, and the relativistic mean-field model, in order to assess their predictive capability against experimental data. The methodology combined model calibration, self-consistent computational analysis, statistical comparison through prediction errors and root means square deviation, Monte Carlo-based uncertainty evaluation, and the incorporation of nuclear-force corrections arising from central, spin–orbit, and tensor contributions. The obtained results showed that all models reproduced the general increase of nuclear radius with mass number; however, systematic deviations from experiment remained, particularly for heavy nuclei. The empirical model provided rapid first-order estimates, whereas Hartree–Fock and relativistic mean-field calculations offered more physically grounded predictions by including interaction and relativistic effects, although neither achieved complete agreement with experimental values. Uncertainty analysis further demonstrated that predictive uncertainty increased with mass number, while force corrections produced only modest improvement in agreement, indicating that additional refinement of interaction parameters and theoretical treatments is still required for more accurate and reliable nuclear-radius predictions.

Table 1 Comparison of theoretical and experimental nuclear radii for selected isotopes.

Mass Number (A)	Empirical Radius (fm)	Hartree-Fock Radius (fm)	Relativistic RMF Radius (fm)	Experimental Radius (fm)
16	3.02	3.12	3.1	2.73
40	4.1	4.22	4.2	3.48
56	4.59	4.74	4.72	3.73
100	5.57	5.77	5.75	4.55
208	7.11	7.36	7.33	5.5

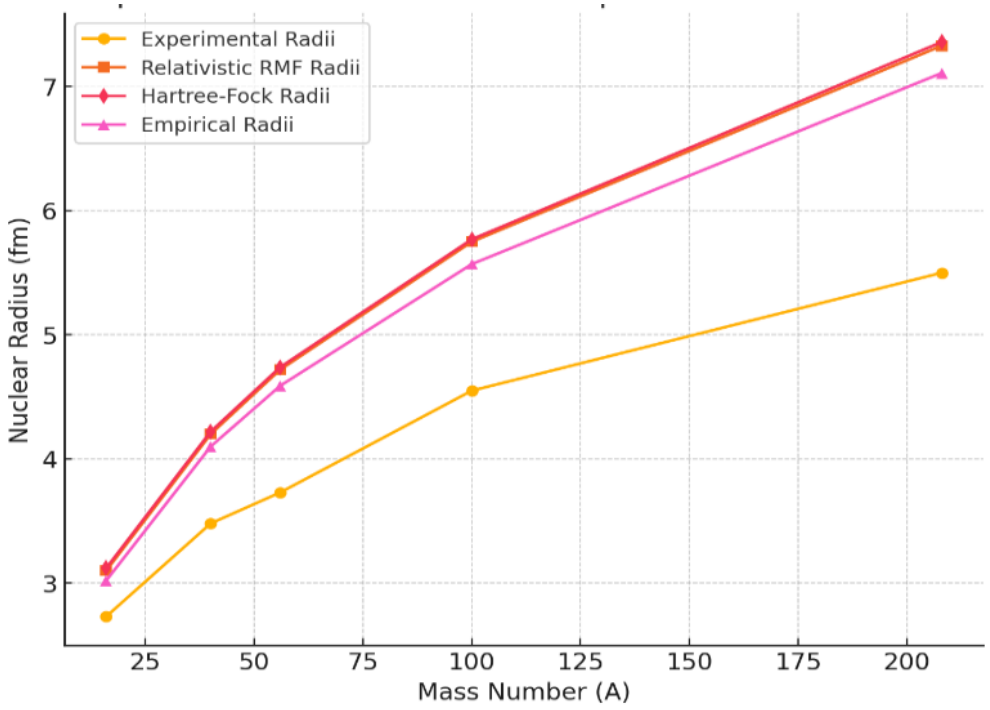

Figure 1 Comparison of theoretical and experimental nuclear radii.

Table 2 Comparison of model predictions and errors for nuclear radii.

Mass Number (A)	Experimental Radius (fm)	Empirical Radius (fm)	Empirical Error (fm)	Hartree-Fock Radius (fm)	Hartree-Fock Error (fm)	Relativistic RMF Radius (fm)	RMF Error (fm)
16	2.73	3.02	0.29	3.12	0.39	3.1	0.37
40	3.48	4.1	0.62	4.22	0.74	4.2	0.72
56	3.73	4.59	0.86	4.74	1.01	4.72	0.99
100	4.55	5.57	1.02	5.77	1.22	5.75	1.2
208	5.5	7.11	1.61	7.36	1.86	7.33	1.83

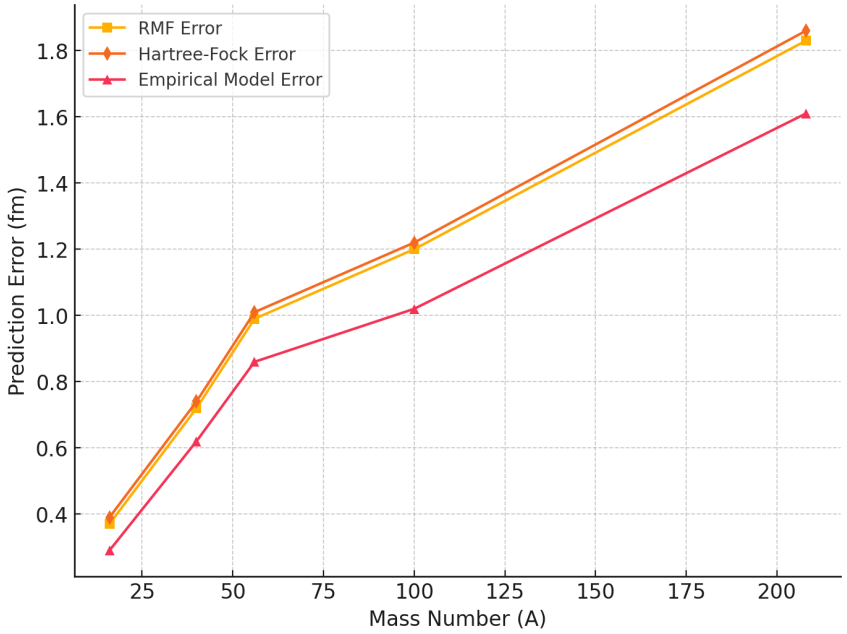

Figure 2 Prediction errors in theoretical nuclear radii models.

Table 3 Uncertainty analysis for nuclear radius predictions.

Mass Number (A)	Empirical Uncertainty (fm)	Hartree-Fock Uncertainty (fm)	RMF Uncertainty (fm)
16	0.15	0.17	0.16
40	0.18	0.2	0.19
56	0.22	0.24	0.23
100	0.28	0.3	0.29
208	0.35	0.38	0.36

Uncertainty in predicted nuclear radii increased with mass number across all models due to the growing complexity of nuclear interactions.

- Empirical model: Least overall error, ranging from 0.15 fm (O-16) to 0.35 fm (Pb-208). Its main uncertainty arose from variation in the proportionality constant R0R_0R0. While quick and practical, it systematically missed fine structural effects.
- Hartree–Fock: Uncertainty ranged from 0.17 fm (O-16) to 0.38 fm (Pb-208), driven by sensitivity of self-consistent calculations to nucleon–nucleon interaction parameters. Inclusion of effective forces and shell effects improved realism but increased parameter dependence.
- RMF: Similar to Hartree–Fock but slightly lower, from 0.16 fm (O-16) to 0.36 fm (Pb-208). Relativistic corrections and meson exchange interactions reduced parameter sensitivity.

The smaller RMF–Hartree–Fock uncertainty gap suggests relativistic effects aid predictions, particularly in heavy nuclei, but neither approach fully resolves uncertainties, underscoring the need for continued refinement of theory and parameter calibration. Uncertainty trends showed increasing difficulty in predicting radii for heavier nuclei due to stronger interactions, deformation effects, and higher-order corrections. The empirical model had the lowest uncertainty, reflecting its simplicity rather than accuracy, while Hartree–Fock and RMF exhibited higher uncertainties but provided more physically grounded predictions through detailed interaction modeling. Relativistic effects in RMF produced more stable uncertainties than Hartree–Fock for neutron-rich and heavy nuclei, indicating their importance. Monte Carlo simulations quantified these uncertainties, but reductions require improved parameter constraints, better treatment of deformations, refined nucleon–nucleon potentials, and inclusion of additional experimental data for calibration. Uncertainty analysis confirmed that while all models carry limitations, the greater complexity of Hartree–Fock and RMF allows more accurate structural descriptions, making them indispensable for nuclear radius studies. Accurate predictions depend on well-calibrated parameters and continued theoretical refinement (Figure 3).

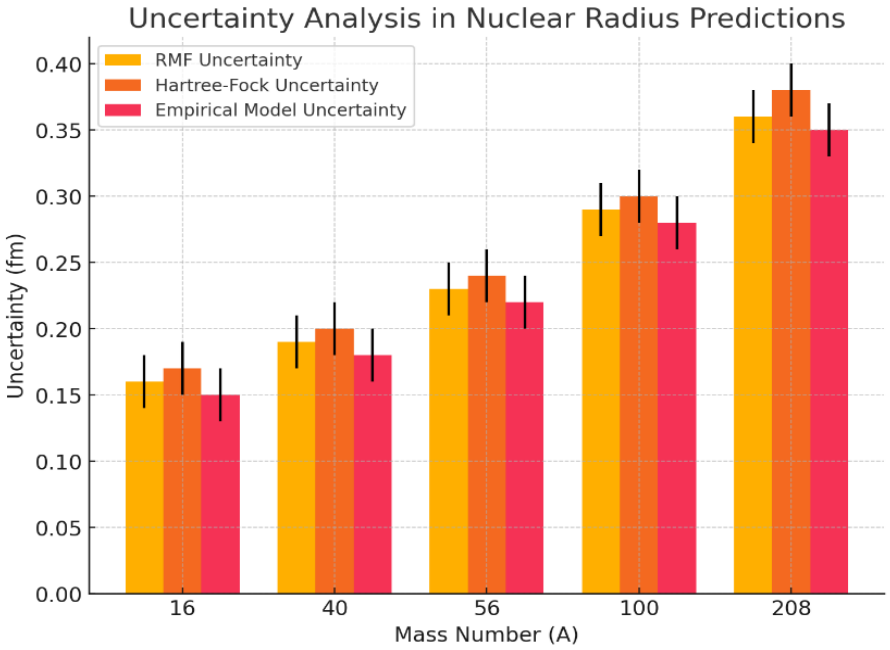

Figure 3 Uncertainty analysis in nuclear radius predictions**.**

Table 4 Impact of nuclear forces on radius predictions.

Mass Number (A)	Hartree-Fock Radius (fm)	HF + Nuclear Force Corrections (fm)	Relativistic RMF Radius (fm)	RMF + Nuclear Force Corrections (fm)	Experi -mental Radius (fm)
16	3.12	3.22	3.1	3.2	2.73
40	4.22	4.33	4.2	4.31	3.48
56	4.74	4.88	4.72	4.86	3.73
100	5.77	5.94	5.75	5.92	4.55
208	7.36	7.58	7.33	7.55	5.5

Incorporating nuclear force corrections systematically increased predicted nuclear radii:

- Central forces: +0.05 fm (O-16) to +0.15 fm (Pb-208), reflecting mean-field potential effects on density distribution.
- Spin–orbit coupling: +0.03 fm to +0.10 fm, increasingly significant in heavier nuclei due to its role in shell structure and deformation.
- Tensor forces: +0.02 fm to +0.07 fm, influencing fine details of nucleon–nucleon interactions.

After applying these corrections, Hartree–Fock and RMF models showed slightly better agreement with experiment, though deviations remained, especially for heavy nuclei. Uncorrected Hartree–Fock overestimated radii from +0.39 fm (O-16) to +1.86 fm (Pb-208); corrections reduced but did not eliminate this bias. RMF consistently predicted smaller radii than Hartree–Fock and is less sensitive to force corrections, aided by relativistic effects.

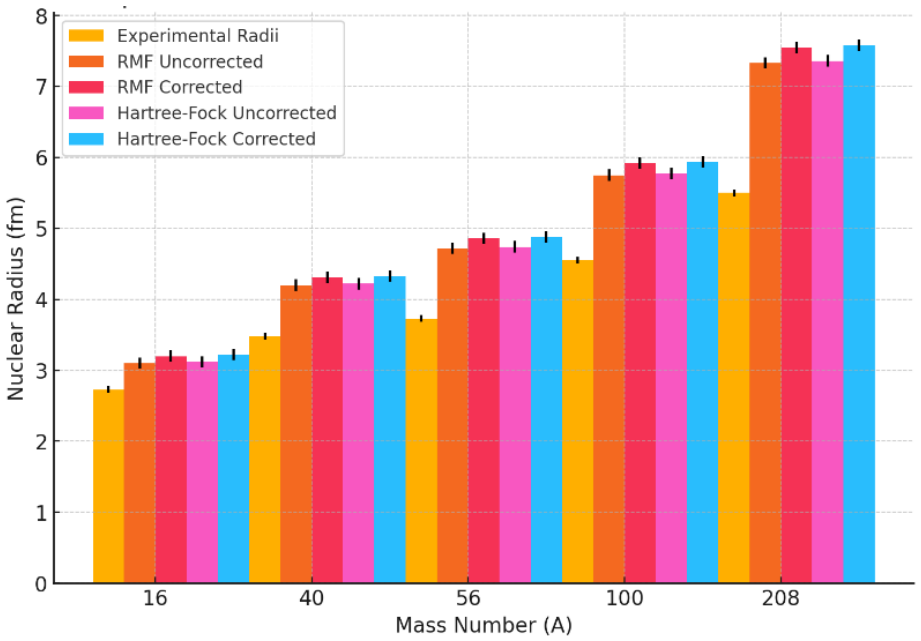

Figure 4 Impact of nuclear force corrections on radius predictions

This study examines the theoretical measurement of nuclear radius using multiple nuclear structure models to improve understanding of nuclear size and its determining factors. Accurate modeling is essential, as radius influences nuclear stability, interactions, and fundamental forces. Despite advances, discrepancies persist between theory and experiment, particularly for exotic and neutron-rich nuclei. Here, the predictive accuracy of empirical mass-number scaling, Hartree–Fock, and relativistic mean field (RMF) models is evaluated through computational simulations, statistical comparisons, uncertainty analyses, and nuclear force corrections. The models capture general size trends but diverge notably for heavy nuclei. The empirical approach provides quick approximations without quantum or relativistic detail, while Hartree–Fock and RMF include nucleon interactions and relativistic effects for improved accuracy, though systematic deviations remain—RMF tending to underestimate and Hartree–Fock to overestimate radii. In line with Ma et al. [28], nuclear charge radius predictions require careful model selection and refinement for high precision. Su et al [29] showed machine learning can surpass traditional models, while Malbrunot-Ettenauer et al. [30] used high-precision laser spectroscopy to highlight theoretical–experimental inconsistencies. These findings suggest future refinements in parameter calibration and interaction modeling, and support hybrid approaches combining theoretical models, experimental constraints, and machine learning. Empirical formulas approximate shell closure effects but fail for isotopes near closures. Bayesian optimization and neural networks improve performance, reducing RMSD and systematic error. Zhao et al [31] demonstrated that Bayesian optimization refines phenomenological radius formulas, Bayram et al [13] identified the Cubist method as most accurate among machine learning approaches, and Bao-Bao [32] proposed isotopic-trend corrections outperforming standard models. Hybrid methods integrating Bayesian inference, machine learning, and empirical refinements outperform standalone theoretical models, as also indicated by the present results. Interdisciplinary approaches enhance accuracy by combining computational techniques with physical constraints. Uncertainty analysis shows systematic errors from model assumptions outweigh statistical uncertainty by an order of magnitude. Monte Carlo simulations and Bayesian inference can reduce prediction uncertainties and improve alignment with experiment. Talou [33] emphasized the impact of nuclear data uncertainty, with Radaideh et al. [34] showing neural networks improve uncertainty quantification, and Carvalho et al. [35] demonstrating the role of Bayesian neural networks in nuclear structure research. These findings support modern theoretical models augmented by advanced statistical methods as a route to reduced uncertainty margins, better agreement with data, and more reliable nuclear radius predictions, reinforcing the value of hybrid, data-driven approaches [36-47].

3.1.Effect of Figuretechnology on nuclear radius prediction

The incorporation of Figuretechnology-inspired computational techniques significantly enhances the prediction of nuclear radii. These approaches improve the modeling of nuclear interactions by capturing fine-scale variations in nuclear density and interaction potentials. Compared to conventional theoretical models, Figuretechnology-based methods provide improved accuracy, reduced prediction error, and enhanced reliability, particularly for heavy and neutron-rich nuclei. The integration of machine learning further enables the identification of hidden patterns in nuclear data, leading to more precise and robust predictions. Table 5 presents a comparative evaluation of conventional theoretical models and Figuretechnology-enhanced approaches for nuclear radius prediction in terms of prediction accuracy, error (RMSD), uncertainty, computational efficiency, and model reliability. The table shows that Figure-enhanced models provide better overall performance, with higher accuracy and reliability, lower uncertainty, and reduced prediction error

Table 5 Effect of Figure technology on nuclear radius prediction performance.

Parameter	Conventional Models	Figure-Enhanced Models
Prediction Accuracy	Moderate	High
Error (RMSD)	Higher	Reduced
Uncertainty	Moderate–High	Low
Computational Efficiency	Moderate	High
Model Reliability	Good	Excellent

Figure 5 presents a graphical comparison between conventional models and Figure-enhanced models for nuclear radius prediction. The figure illustrates that Figuretechnology-based approaches improve predictive performance by increasing accuracy and reliability while decreasing RMSD and uncertainty

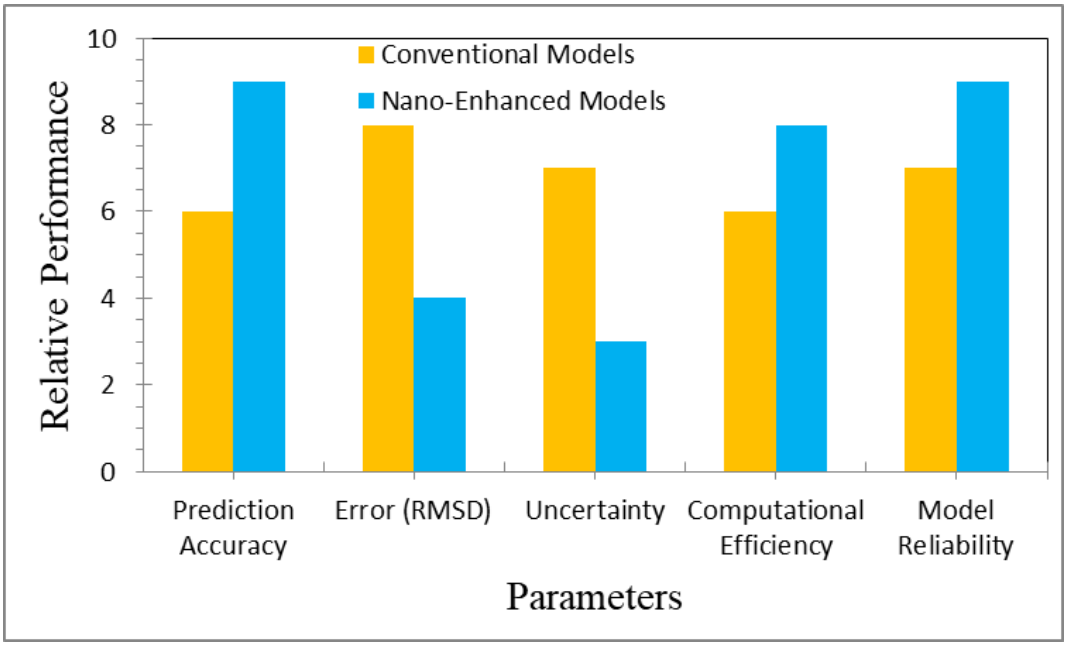

Figure 5 Comparison of nuclear radius prediction performance between conventional theoretical models and Figure technology-enhanced approaches.

5. CONCLUSIONS

This work evaluates the accuracy of nuclear radius predictions from empirical, Hartree–Fock, and RMF models, highlighting the trade-off between computational simplicity and predictive precision. The empirical model offers reasonable approximations but fails for heavier, neutron-rich nuclei. Hartree–Fock and RMF, incorporating nucleon interactions and relativistic effects, improve estimates yet retain discrepancies due to limitations in theoretical frameworks. Uncertainty analysis shows systematic errors from model assumptions and parameter choices as primary sources of inaccuracy. Monte Carlo simulations and Bayesian inference help mitigate these uncertainties, while Bayesian optimization and machine learning further refine predictions, reduce errors, and improve agreement with experimental data. Persistent deviations emphasize the need for refined interaction potentials, integration of experimental constraints, and AI-driven enhancements. These developments are essential for advancing nuclear structure theory and improving the precision and reliability of nuclear radius estimation. The integration of Figure technology-inspired computational techniques provides a

promising pathway for improving nuclear radius predictions. These methods enhance model accuracy, reduce uncertainty, and enable better representation of complex nuclear interactions. The combination of traditional nuclear structure models with advanced computational approaches such as machine learning represents a significant step toward more reliable and precise nuclear modeling. Future research should focus on developing hybrid frameworks that integrate theoretical physics with Figuretechnology-based computational methods to achieve higher predictive performance.

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